**Intuitive Class/Shape Function Parameterization for Airfoils**

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| **ABSTRACT** | Haut du formulaire    Bas du formulaire | [Next section](https://arc.aiaa.org/doi/10.2514/1.J052610#_i2) |

An airfoil parameterization method is proposed, which combines the flexibility and accuracy of Kulfan’s class/shape function transformation method and the intuitiveness of Sobieczky’s parameterization method for airfoil sections. The proposed intuitive class/shape function transformation method has been evaluated by comparing it with the class/shape function transformation and parameterization method for airfoil sections regarding their accuracy in inversely fitting a wide range of airfoils. The results show that the intuitive class/shape function transformation method is able to successfully transform the class/shape function transformation method into a fully intuitive parametric method (in which all of the design variables are aerodynamically related geometrical parameters) without loss of the accuracy and flexibility of the class/shape function transformation method for the airfoils tested.

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| **I. Introduction** | Haut du formulaire    Bas du formulaire | [Previous section](https://arc.aiaa.org/doi/10.2514/1.J052610#abstract)[Next section](https://arc.aiaa.org/doi/10.2514/1.J052610#_i3) |

Shape parameterization, that is the representation of a set of geometries by a number of design variables, is a crucial step in aerodynamic optimization. The parameterization has a profound effect on the design space, which determines the optimum geometries obtainable. Therefore, it is a key decision in the numerical optimization process for the designer to choose a proper shape parameterization method.

There are a number of shape parameterization methods available in aerodynamic optimization. Samareh [[1](https://arc.aiaa.org/doi/10.2514/1.J052610)] reviewed and compared some of these methods and classified the shape parameterization methods into eight categories, that is, the basis vector, domain element, partial differential equation, discrete (mesh point), polynomial and spline, analytical, computer-aided design (CAD)-based, and free-form deformation methods.

A well-behaved parameterization method should have the following properties [[1](https://arc.aiaa.org/doi/10.2514/1.J052610),[2](https://arc.aiaa.org/doi/10.2514/1.J052610)]: 1) high flexibility to cover the potential optimal solution in the design space, 2) small number of key design variables, 3) smoothness and realizability of the shapes, and 4) intuitiveness of the design parameters for geometrical and physical understanding by the design engineers in exploring the design space and setting up optimization constraints. In actual applications, a balance needs to be met for parameterization, as it is unlikely that all the requirements can be satisfied. For example, the mesh point method is the simplest and the most flexible method, which directly uses the grid points as the design variables [[3–6](https://arc.aiaa.org/doi/10.2514/1.J052610)] and is theoretically able to represent any geometry as long as there are sufficient surface points. However, it results in a very large number of design variables, and the intuitive bounds, such as leading-edge radius, are difficult to set. It also increases the complexity of the optimization problem. With a large number of design variables, in which the adjoint methods are necessary for numerical efficiency to calculate the sensitivities in gradient-based optimization [[4](https://arc.aiaa.org/doi/10.2514/1.J052610),[5](https://arc.aiaa.org/doi/10.2514/1.J052610),[7](https://arc.aiaa.org/doi/10.2514/1.J052610),[8](https://arc.aiaa.org/doi/10.2514/1.J052610)], the smoothness of the resulting shape and noise in design space are still challenging issues.

The Bezier polynomial and B-spline methods are widely used parameterization methods in aerodynamic design parameterization [[1](https://arc.aiaa.org/doi/10.2514/1.J052610),[6](https://arc.aiaa.org/doi/10.2514/1.J052610),[7](https://arc.aiaa.org/doi/10.2514/1.J052610),[8–10](https://arc.aiaa.org/doi/10.2514/1.J052610)]. Various authors have presented the applications of the Bezier polynomial, B-spline, and Non-Uniform Rational B-Spline methods for three-dimensional aerodynamic optimization [[11–16](https://arc.aiaa.org/doi/10.2514/1.J052610)]. In recent years, CAD-based methods and free-form deformation methods are becoming increasingly popular [[6](https://arc.aiaa.org/doi/10.2514/1.J052610),[17–23](https://arc.aiaa.org/doi/10.2514/1.J052610)]. These shape parameterization methods have excellent performance in terms of properties 1–3. However, intuitive parameters are absent.

Intuitive parameters can assist the designer to understand shape features and to directly influence the design. Furthermore, in a shape design optimization process, the constraints can be simplified and directly set. On the other hand, in the nonintuitive parameterization methods, the bounds of design variables are decided by the users more in a heuristic way and are often either overlimiting or underlimiting the design space. To satisfy the constraints in a nonintuitive parameterization, extra functions are required to calculate the geometric constraints, such as the thickness, the leading-edge radius, the trailing-edge angle, and so on. This would increase the complexity of optimization, for example for gradient-based optimization, because these constraint functions are not necessarily differentiable, and in some cases, reduce the efficiency of the optimizer. In intuitive parameterization methods, the bound of the design variable could be easily selected, because the design variables are understood by the users, and some intuitive design variables are explicitly linked with geometric constraints.

The parameterization method for airfoil sections (PARSEC) is one of the most popular intuitive airfoil parameterization methods proposed by Sobieczky [[24](https://arc.aiaa.org/doi/10.2514/1.J052610)], which provides a set of full intuitive parameters for airfoil representation and has been used for airfoil optimization. The method satisfies the aforementioned properties 2–4. But its flexibility is questionable as discussed and demonstrated later in this paper.

More recently, Kulfan [[2](https://arc.aiaa.org/doi/10.2514/1.J052610),[25–27](https://arc.aiaa.org/doi/10.2514/1.J052610)] developed the class/shape function transformation (CST) parameterization method, which is more flexible, being able to produce a wider range of shapes, including airfoils, in a universal way, and including a few intuitive parameters, such as an airfoil leading-edge radius and its trailing-edge vertical position. The method has a reasonably small number of design variables, depending on the order of the polynomials used. Similar to other polynomial-based parameterizations mentioned earlier, property 4 is not satisfied.

A few researchers have compared the performance of CST and PARSEC methods with other shape parameterization methods. Sripawadkul and Padulo [[28](https://arc.aiaa.org/doi/10.2514/1.J052610)] studied and compared five airfoil parameterization methods, namely, Ferguson’s curves, Hicks–Henne bump functions, B-spline, PARSEC, and the CST transformation method, in terms of parsimony, completeness, orthogonally, flawlessness, and intuitiveness. Five parameterization methods were scored to assist in selecting the proper method in respect to specific issues. Castonguay and Nadarajah [[3](https://arc.aiaa.org/doi/10.2514/1.J052610)] studied the effect of four parameterization methods, that is, the mesh points, B-spline, Hicks–Henne bump function, and PARSEC methods, on inverse design and drag minimization for two-dimensional airfoils. The results demonstrated that the mesh points method provides the highest level of accuracy/flexibility compared to the other methods, and PARSEC is unable to provide high flexibility because it failed in the inverse design case for some airfoils. Mousavi et al. [[6](https://arc.aiaa.org/doi/10.2514/1.J052610)] performed the two-dimensional airfoil inverse design, two-dimensional drag minimization, and three-dimensional wing drag minimization using the mesh points, B-spline, and CST methods. It showed that the mesh point methods provided the best results in all test cases. The B-spline and CST methods were able to provide reasonable flexibility with a low number of design variables. The CST method was able to eliminate the shock wave using a very low number of variables in the drag minimization case. Paul and Ruxandra [[29](https://arc.aiaa.org/doi/10.2514/1.J052610)] compared the mesh point, polynomial, Bezier curve, B-spline, and CST methods for low-speed airfoil designs. They concluded that although B-spline could achieve better optimal results than the other methods, the Bezier curve and CST method have better performance when designing a completely new airfoil.

The above review shows that the CST method can provide relatively high flexibility with a reasonably low number of design variables with a couple of intuitive parameters. Although PARSEC is able to provide a full set of intuitive parameters, it is unable to provide the required flexibility for airfoil design. In this work, a new airfoil parameterization method, named the intuitive CST or intuitive class/shape function transformation (iCST) method, is proposed to combine the advantages of both CST and PARSEC methods. Its performance is evaluated by comparing with the CST and PARSEC methods in terms of the accuracy in inverse curve fitting for a range of airfoils.

**A. PARSEC Method**

The PARSEC parameterization method is developed by Sobieczky [[24](https://arc.aiaa.org/doi/10.2514/1.J052610)]. The purpose of the method is to find a minimum number of variables to address the special aerodynamic, geometric, and flow features. In this method, 11 intuitive parameters are employed to explicitly represent an airfoil as showed in Fig. [1](https://arc.aiaa.org/doi/10.2514/1.J052610). They are the leading-edge radius (RleRle), upper crest position (XupXup, ZupZup), upper crest curvature (ZxxupZxxup), lower crest position (XloXlo, ZloZlo), lower crest curvature (ZxxloZxxlo), trailing-edge position (ZteZte), trailing thickness (ΔZteΔZte), and trailing-edge angle and trailing-edge wedge angle (αteαte and βteβte).

**B. CST Method**

The CST method, proposed by Kulfan [[2](https://arc.aiaa.org/doi/10.2514/1.J052610),[25–27](https://arc.aiaa.org/doi/10.2514/1.J052610)], is increasingly used in airfoil optimization. The purpose of this method is to develop a universal parameterization for complex aircraft configuration, which is not limited only to airfoils. For the normal airfoil with a round nose leading edge and a boat-tail trailing edge, the difficulties in representing it mathematically are due to the infinite slope and second derivative requirement at the leading edge and large variations of curvature over the shape. The CST method was intended to overcome these limits and represent the different types of geometries in a generic way.